Functions – Rational Functions

Looking at the functions $f(x) = \frac{x^2-2x-8}{x+1}$, there is a vertical asymptote at $x = -1$.

The degree of the numerator (2) is greater than the degree of the denominator (1),

So there is NO horizontal asymptote.

But this function does have ANOTHER asymptote.

When the degree of the numerator is 1 greater than the degree of the denominator,

There is a SLANT ASYMPTOTE (also called an OBLIQUE ASYMPTOTE).

This is an asymptote that goes on an angle.

Ex: $f(x) = \frac{x^2-2x-8}{x+1}$. To find the slant asymptote, you must use long division.

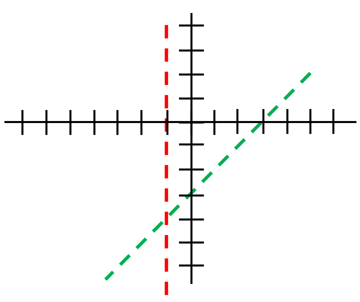
$$
\begin{array}{r}
x - \ \ 3 \\
x+1 \overline{\big)\ x^2 - 2x - 8} \\
-\ \underline{x^2 + \ x \quad} \\
-3x - 8 \\
-\ \underline{-3x - 3} \\
-5
\end{array}
$$

The quotient is the slant asymptote:

$y = x - 3$

To graph this function, plot the asymptotes first.

Note there is also a vertical asymptote at $x = -1$

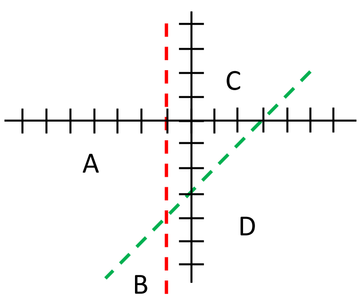


← $x = -1$ ← $y = x - 3$

Then evaluate the function to the left and right of the vertical asymptote.

$f(-2) = \frac{(-2)^2-2(-2)-8}{(-2)+1} = \frac{4+4-8}{-1} = \frac{0}{-1} = 0$

$f(0) = \frac{(0)^2-2(0)-8}{(0)+1} = \frac{0+0-8}{1} = \frac{-8}{1} = -8$



The point $(-2, 0)$ is in the region labeled “A”.

The point $(0, -8)$ is in the region labeled “D”.

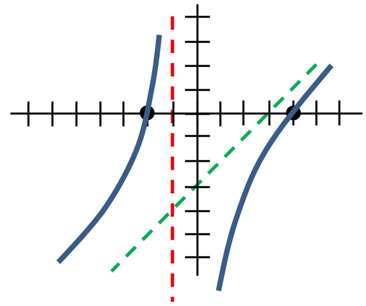
There are x-intercepts (numerator = 0)

$x^2 - 2x - 8 = (x-4)(x+2) = 0$

So the points $(4, 0)$ & $(-2, 0)$ are on the graph.

So a sketch of the function looks

something like:



Holes:

Graph the function $(x) = \frac{x-2}{x^2+x-6}$.

The denominator has a larger degree than the numerator, so there is a horizontal asymptote: $y = 0$.

The denominator factors like this: $x^2 + x - 6 = (x+3)(x-2) = 0$, so there appears to be two vertical asymptotes: $x = -3$ & $x = 2$.

BUT NOTICE that this function can be simplified: $\frac{x-2}{x^2+x-6} = \frac{x-2}{(x+3)(x-2)} = \frac{1}{(x+3)}$

So is there a problem at $x = 2$, or not? There was at the start, but it went away.

We say there is a removable discontinuity at $x = 2$, because simplifying the function "removed" the behavior that broke the graph apart at $x = 2$.

So there is ONLY 1 vertical asymptote for this function: $x = -3$.

But x can still not be equal to 2, because the unsimplified function is still undefined there.

In this case, there is no vertical asymptote at $x = 2$, but there is a HOLE there.

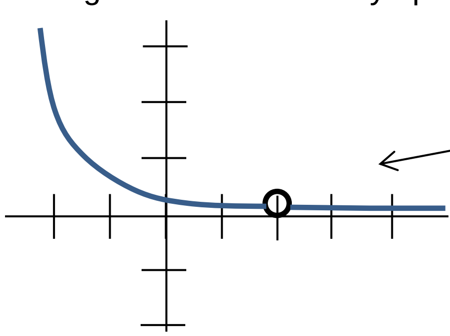
The resulting graph looks like the graph of the simplified function $f(x) = \frac{1}{x+3}$.

At $x = 2$, $f(2) = \frac{1}{2+3} = \frac{1}{5}$, so the hole will be at $(2, \frac{1}{5})$.

The function $f(x) = \frac{1}{x+3}$, has a vertical asymptote at $x = -3$,
and a horizontal asymptote at $y = 0$.

Left of the vertical asymptote: $f(-4) = \frac{1}{(-4)+3} = \frac{1}{-1} = -1$

Right of the vertical asymptote: $f(0) = \frac{1}{0+3} = \frac{1}{3}$



But if you looked closely at the graph at x =2, you'd see that there is a hole in the graph there.